Section 9.1
Direct, Inverse and Joint Variation

Direct Variation

Indirect Variation

Joint Variation

The relationship between two variables (x and y) varies according to a function with a constant value k.
The constant, k, is called the constant of variation.

Basic Variation equations

Direct Variation (both increase/decrease)

$$y = kx$$

Indirect Variation (one goes up the other down)

$$y = \frac{k}{x}$$

Joint Variation (varies with two or more variables)

$$y = kxz$$

Solving Variation Problems

1. Write an equation that describes the given English statement.
2. Substitute the given pair of values into the equation in step 1 and solve for k, the constant of variation.
3. Substitute the value of k into the equation in step 1.
4. Use the equation from step 3 to answer the problem's question.

Direct Variation with Powers

y* varies directly as the *n*th power of *x if there exists some nonzero constant k such that

$$y = kx^n.$$

We also say that ***y* is directly proportional to the *n*th power of *x***.

General Variation equations

Direct Variation

$$y = kx \quad \text{or} \quad y = kx^n$$

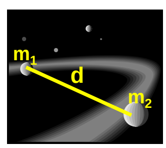
Indirect Variation

$$y = \frac{k}{x} \quad \text{or} \quad y = \frac{k}{x^n}$$

Joint Variation

$$y = kxz \quad \text{or} \quad y = k\frac{x}{z} \quad \text{or} \quad y = kx^n z^n \quad \text{or} \quad y = k\frac{x^n}{z^n}$$

Joint variation is a variation in which a variable varies directly as the product of two or more other variables. Thus, the equation $y = kxz$ is read "y varies jointly as x and z."
Joint variation plays a critical role in Isaac Newton's formula for gravitation:

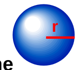


$$F = G\frac{m_1 m_2}{d^2}$$

The formula states that the force of gravitation, F, between two bodies varies jointly as the product of their masses, m_1 and m_2, and inversely as the square of the distance between them d. G is the gravitational constant. The formula indicates that gravitational force exists between any two objects in the universe, increasing as the distance between the bodies decreases.

D.V. Example

The volume of a sphere varies directly as the cube of the radius. If the volume of a sphere is ~523.6 or (500/3)π cubic inches when the radius is 5 inches, what is the radius when the volume is 33.5 or (32/3)π cubic inches.



Equation for volume of a sphere is $V = kr^3 = \frac{4}{3}\pi r^3$

$V_1 = \frac{500}{3}\pi$ $\quad$ $V_2 = \frac{32}{3}\pi$ $\quad$ $r_2 = 2$

$r_1 = 5$ $\quad$ $r_2 = ?$

I.V. Example

The pressure, P, of a gas in a spray container varies inversely as the volume, V, of the container. If the pressure is 6 pounds per square inch when the volume is 4 cubic inches, what is the volume when the pressure is down to 3 pounds per square inch?

Equation for pressure (P) is $P = \frac{k}{V} = \frac{k}{\pi r^2 h}$

$P_1 = \frac{k}{V_1}$ or $P_1V_1 = k$ $\quad$ $(6)(4) = k, \quad 24 = k$

$P_2V_2 = k$ $\quad$ $(3)(V_2) = 24$ $\quad$ $V_2 = 8$

Joint Variation

We can combine two variation equations:

$S = kA$

which says that the monthly sales, S, varies directly as the advertising budget A and

$S = \frac{k}{P}$

which says that the monthly sales, S, vary inversely as the price of the product, P.

$S = k\frac{A}{P}$

J.V. Example

The TIXY calculator leasing company has determined that leases L, vary directly as its advertising budget and inversely as the price/month of the calculator rentals. When the TIXY company spent $500 on advertising on the internet and charge $30/month for the rentals, their monthly rental income was $4000. Write an equation of variation that describes this situation. Determine the monthly leases if the amount of advertising is increased to $2000.

$L = k_1 A$ and $L = \frac{k_2}{P}$ so $L = \frac{k_T A}{P}$ and $\frac{\text{LP}}{A} = k_T$

$L_1 = 4000$, $A_1 = 500$, $P_1 = 30$

$\frac{\text{LP}}{A} = k_T$

$\frac{(4000)(30)}{500} = k_T$

$240 = k_T$

$L_2 = ?$, $A_2 = 2000$, $P_2 = 30$

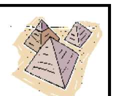
$L_2 = \frac{k_T A_2}{P_2}$

$L_2 = \frac{(240)(2000)}{30}$

$L_2 = \frac{(240)(2000)}{30}$

$L_2 = 16000$

J. V. Example



The volume of a model square based pyramid, V, various jointly as its height, h, and the square of its side, s ,of the square base. A model pyramid that has a side of the square base that is 6 inches, and the height is 10 inches, has a volume of 120 cubic inches. Find the volume of a pyramid with a height of 9 inches and a square base of 5 inches.

$V = khs^2$ and $k = \frac{V}{hs^2}$

$V_1 = 120$, $h_1 = 10$, $s_1 = 6$

$k = \frac{V_1}{h_1 {s_1}^2} = \frac{120}{(10)(6^2)} = \frac{120}{360} = \frac{1}{3}$

$V_2 = ?$, $h_2 = 9$, $s_2 = 5$

$V_2 = kh_2 {s_2}^2 = \frac{1}{3}(9)(5^2) = 75$