A **polyhedron** is a solid that is bounded by polygons, called **faces**, that enclose a single region of space.

An **edge** of a polyhedron is a line segment formed by the intersection of two faces.

A **vertex** of a polyhedron is a point where three or more edges meet.

The plural of polyhedron is **polyhedra**, or polyhedrons.
Of the five solids below, the prism and the pyramid are polyhedra. The cone, cylinder, and sphere are not polyhedra.
Notice that the sum of the number of faces and vertices is two more than the number of edges. This result was proved by the Swiss mathematician Leonhard Euler (1707 - 1783).

**THEOREM**

**Theorem 12.1  Euler’s Theorem**

The number of faces \( F \), vertices \( V \), and edges \( E \) of a polyhedron are related by the formula \( F + V = E + 2 \).
There are five regular polyhedra, called *Platonic solids*, after the Greek mathematician and philosopher Plato.

The **Platonic solids** are a regular **tetrahedron** (4 faces), a **cube** (6 faces), a regular **octahedron** (8 faces), a regular **dodecahedron** (12 faces), and a regular **icosahedron** (20 faces).
Identifying Polyhedra

Decide whether the solid is a polyhedron. If so, count the number of faces, vertices, and edges of the polyhedron.

SOLUTION

This is a polyhedron. It has 5 faces, 6 vertices, and 9 edges.
Identifying Polyhedra

Decide whether the solid is a polyhedron. If so, count the number of faces, vertices, and edges of the polyhedron.

SOLUTION

This is a polyhedron. It has 5 faces, 6 vertices, and 9 edges.

This is not a polyhedron. Some of its faces are not polygons.
Identifying Polyhedra

Decide whether the solid is a polyhedron. If so, count the number of faces, vertices, and edges of the polyhedron.

SOLUTION

This is a polyhedron. It has 5 faces, 6 vertices, and 9 edges.

This is not a polyhedron. Some of its faces are not polygons.

This is a polyhedron. It has 7 faces, 7 vertices, and 12 edges.
Of the five solids below, the prism and the pyramid are polyhedra. The cone, cylinder, and sphere are not polyhedra.
A polyhedron is **regular** if all of its faces are congruent regular polygons.

A polyhedron is **convex** if any two points on its surface can be connected by a segment that lies entirely inside or on the polyhedron.

If this segment goes outside the polyhedron, then the polyhedron is **nonconvex**, or **concave**.

![Regular convex polyhedron](image1.png)

regular, convex

![Nonregular nonconvex polyhedron](image2.png)

nonregular, nonconvex
Is the octahedron convex? Is it regular?

convex, regular
Is the octahedron convex? Is it regular?

- convex, regular
- convex, nonregular
Is the octahedron convex? Is it regular?

- convex, regular
- convex, nonregular
- nonconvex, nonregular
Imagine a plane slicing through a solid.

The intersection of the plane and the solid is called a **cross section**.

For instance, the diagram shows that the intersection of a plane and a sphere is a circle.
Describe the shape formed by the intersection of the plane and the cube.

**SOLUTION**

This cross section is a square.
Describing Cross Sections

Describe the shape formed by the intersection of the plane and the cube.

SOLUTION

This cross section is a square.

This cross section is a pentagon.
Describe the shape formed by the intersection of the plane and the cube.

SOLUTION

This cross section is a square.

This cross section is a pentagon.

This cross section is a triangle.
Describe the shape formed by the intersection of the plane and the cube.

**SOLUTION**

This cross section is a square.

This cross section is a pentagon.

This cross section is a triangle.

The square, pentagon, and triangle cross sections were just described above. Some other cross sections are the rectangle, trapezoid, and hexagon.
There are five regular polyhedra, called **Platonic solids**, after the Greek mathematician and philosopher Plato.

The **Platonic solids** are a regular tetrahedron (4 faces), a cube (6 faces), a regular octahedron (8 faces), a regular dodecahedron (12 faces), and a regular icosahedron (20 faces).
Notice that the sum of the number of faces and vertices is two more than the number of edges in the last slide. This result was proved by the Swiss mathematician *Leonhard Euler* (1707 - 1783).

**THEOREM 12.1  Euler’s Theorem**

The number of faces \((F)\), vertices \((V)\), and edges \((E)\) of a polyhedron are related by the formula \(F + V = E + 2\).
The solid has 14 faces: 8 triangles and 6 octagons. How many vertices does the solid have?

**SOLUTION**

On their own, 8 triangles and 6 octagons have $8(3) + 6(8)$, or 72 edges.

In the solid, each side is shared by exactly two polygons. So, the number of edges is one half of 72, or 36.

Use **Euler’s Theorem** to find the number of vertices.

$$F + V = E + 2$$

$$14 + V = 36 + 2$$

$$V = 24$$

The solid has 24 vertices.
CHEMISTRY  In molecules of sodium chloride, commonly known as table salt, chloride atoms are arranged like the vertices of regular octahedrons. In the crystal structure, the molecules share edges. How many sodium chloride molecules share the edges of one sodium chloride molecule?

SOLUTION  To find the number of molecules that share edges with a given molecule, you need to know the number of edges of the molecule.

You know that the molecules are shaped like regular octahedrons. So, they each have 8 faces and 6 vertices. You can use Euler’s Theorem to find the number of edges, as shown below.

\[ F + V = E + 2 \]

Write Euler’s Theorem.

\[ 8 + 6 = E + 2 \]

Substitute.

\[ 12 = E \]

Simplify.

So, 12 other molecules share the edges of the given molecule.
SPORTS  A soccer ball resembles a polyhedron with 32 faces; 20 are regular hexagons and 12 are regular pentagons. How many vertices does this polyhedron have?

SOLUTION

Each of the 20 hexagons has 6 sides and each of the 12 pentagons has 5 sides.

Each edge of the soccer ball is shared by two polygons. Thus, the total number of edges is as follows:

\[ E = \frac{1}{2} (6 \cdot 20 + 5 \cdot 12) \]

\[ = \frac{1}{2} (180) \]

\[ = 90 \]

Expression for number of edges

Simplify inside parentheses.

Multiply.
**SPORTS** A soccer ball resembles a polyhedron with 32 faces; 20 are regular hexagons and 12 are regular pentagons. How many vertices does this polyhedron have?

**SOLUTION**

Knowing the number of edges, 90, and the number of faces, 32, you can apply Euler’s Theorem to determine the number of vertices.

\[ F + V = E + 2 \]

\[ 32 + V = 90 + 2 \]

\[ V = 60 \]

So, the polyhedron has 60 vertices.